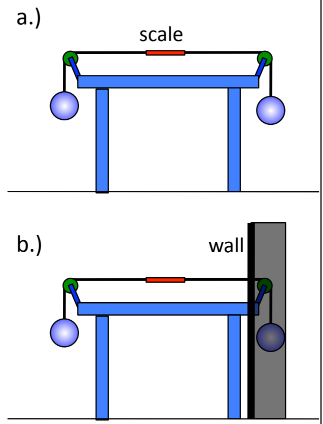
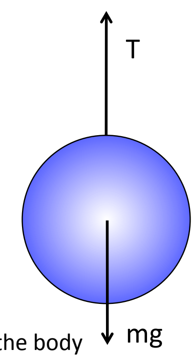
Problem 5.20

In each case, what does the scale read?

a.) This, on the surface, looks complicated. In fact, they are just being tricky in the sense that there is no difference between *Part a* and *Part b* (in both cases, something is holding the ball on the left in equilibrium—it doesn't matter whether it's a connection to a wall or a connection to another hanging mass, the situation is essentially the same). In any case, all the scale is doing is acting like an extension of the string, which means the tension on one side will be the same as the tension on the other. With ideal pulleys and with the mass in equilibrium, we can get that tension by examining either hanging mass (we'll do the one on the left) using N.S.L.



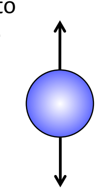
Starting with a free body diagram, see the sketch to the right:



There are three things to notice about this:

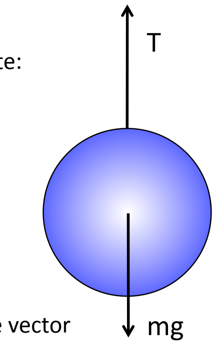
1.) Being in equilibrium, the tension and gravitational forces will have to cancel. That means their magnitudes will be the same. I drew the two force vectors to be the same length. I DIDN'T HAVE TO DO THIS. Free body diagrams conserve orientation of vectors, but they are not necessarily drawn to scale magnitude-wise.

2.) I presented the forces acting *where they are really acting* on the body (that is, gravity is acting at the body's center of mass). This didn't have to be the case. I could as easily have drawn it like the mini-sketch example below. Force *orientation* will always matter, but force placement won't until we begin to deal with torques and rotational motion.



3.) I used a "T" to denote the magnitude of the tension force and an "mg" for the magnitude of the gravitational force. You may run into an alternate version of F_T and F_{mg}. Although your book uses the former, use the latter if your teachers instructs you to do so.

With the free body diagram, we are ready to use N.S.L. and write:



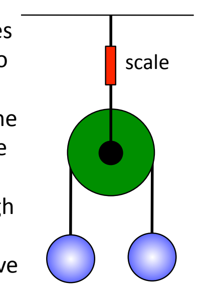
$$\underline{\sum F_y :}$$
$$T - mg = \cancelto{0}{ma_y}$$
$$\Rightarrow \quad T = mg$$
$$\Rightarrow \quad T = (5.00 \text{ kg})(9.80 \text{ m/s}^2)$$
$$= 49.0 \text{ N}$$

Note that each of the algebraic terms (the T and the mg) were vector MAGNITUDES. As such, the signs of those vectors had to be placed manually into the N.S.L. expression. That is why you see a negative sign in front of the "mg" term. That vector is directed downward *in the negative direction*, but because "g" was defined as 9.8 m/s/s (versus -9.8 m/s/s), the unembedded negative sign is needed. Know that ALL TERMS in N.S.L. expressions should be magnitudes.

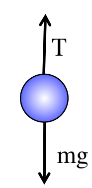
b.) As was pointed out earlier, the f.b.d. we generated for *Part a* works for *Part b*, and the scale reading will be the same, also.

c.) *Part c* is a little tricky for its own reasons. Each of the masses is held up by a rope similarly to *Parts a and b*, but there are two of them being supported by a third rope that the scale is attached to, so the net effect is for the scale to register twice the 49.0 newtons that were generated in *Parts a and b*. That is, the answer is 98.0 newtons. (Assuming you aren't asked to derive results, what I've just stated is the way you should think through a problem, if you can, on an AP test.)

To be complete, I have included the f.b.d.'s you'd need to solve for the scale value using the formal approach to N.S.L.

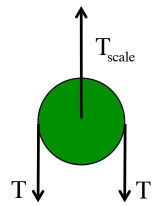


In the formal approach, it is always best to start (and finish off) one object in the system before messing with the second (or third). As such, I will start with a f.b.d. of the ball, then write out N.S.L. for that situation. That is:



$$\underline{\sum F_y :}$$
$$T - mg = \cancelto{0}{ma_y}$$
$$\Rightarrow \quad T = mg = 49.0 \text{ N}$$

Now for the pulley: Noting that ideal pulleys simply redirect the line of tension (that is, the tension magnitude is the same on either side of the pulley), and additionally noticing that the pulley was said to be massless (so there is no "mg" term acting at its center of mass), we can draw the f.b.d. shown to the right, then write:



$$\underline{\sum F_y :}$$

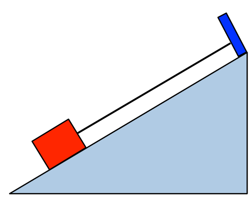
$$-T - T + T_{scale} = \cancelto{0}{m_p a_y}$$

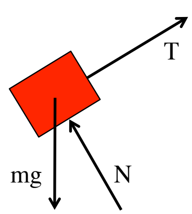
$$\Rightarrow \quad T_{scale} = 2T$$

Solving the ball's equation and the pulley's equation simultaneously yields a scale value of 98 newtons, as expected. Again, in general and especially when taking an AP test, if you are NOT specifically told to derive your results, use any *thinking* approach you can to get your answers.

BIG NOTE: Because the ball part of this problem suggests that "T = mg," students sometimes get it into their heads that "T" is <u>always</u> equal to "mg." Don't do that: THAT IS NOT ALWAYS THE CASE!!!!!

d.) What do we know about a tension force? It will always be directed AWAY FROM the object experiencing it, and it's algebraic symbol is "T." What about gravity? It is always directed toward the center of the earth and AS LONG AS YOU ARE CLOSE TO THE EARTH'S SURFACE its magnitude is "mg." We are about to run into another of the five standard force you'll be working with in this section. That force is called a NORMAL FORCE, and it is a force of support that is ALWAYS PERPENDICULAR to the surface that provides it.

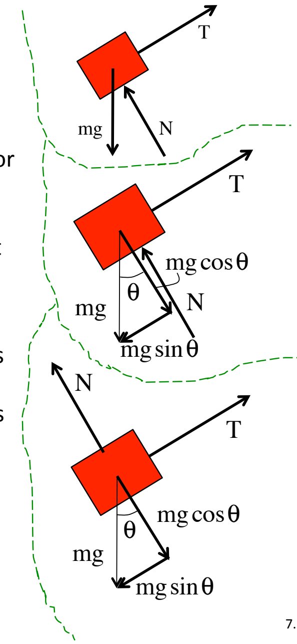




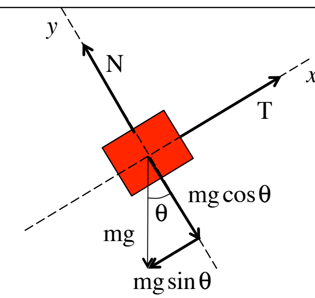
Looking at the f.b.d. on the block, TENSION and GRAVITY make sense. The additional force is a force of support, a NORMAL FORCE, provided by the incline, as shown.

The f.b.d. to the right is technically correct, but at some point we are going to want to break the gravitational force "mg" into its components along the incline and perpendicular to the incline. Why? Although other problems will have better reasons for doing this (I'll point them out when we get there), for this problem we want to know gravitational component that the tension "T" is counteract. That sought-after force is equal to the component of gravity along the incline.

The problem, as you can see in the second sketch, is that with the normal force in it's technically correct position, things get cluttered. The solution, and this is perfectly acceptable to do in problem in which torques are not being dealt with, is to slide the normal force into the position shown in the last sketch. In that way, we can easily see and deal with the needed components of gravity, and we haven't compromised the orientation of any of the vectors.



So using that last f.b.d. and adding a coordinate system whose axes are *up the incline* (the x-direction) and perpendicular to *up the incline* (the y-direction), we can write:



$$\sum F_x :$$

$$T - mg\sin\theta = \cancelto{0}{ma_x}$$

$$\Rightarrow \quad T = mg\sin\theta$$

$$= (5.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 30^\circ$$

$$= 24.5\ \text{N}$$

The only additional bit of nastiness is in justifying that the angle used to determine gravity's components is really the angle of the incline. A proof of that is shown on the next page. You don't have to read this is you believe the angle is as presented. The proof is provide, though, with the understanding that once done, we will know that the *angle of the incline* will ALWAYS BE THE SAME AS the *angle between* the vertical and the normal on an incline.

Specifically: Look at the sketch. The right triangle has an angle θ and an angle $90^\circ - \theta$ in it, where the latter angle is bordered by a line in the vertical.

Now notice that the *line of the incline* and the *line of the normal* are at right angles to one another (this is denoted by the small red square).

It follows that the angle between the normal and the vertical is θ, as shown.